I

APAE MEMO NO. 96
COPY NO. 19

APPR-1 HOT CHANNEL FACTORS

RE-EVALUATION ON THE BASIS OF MANUFACTURING EXPERIENCE AND ZERO POWER EXPERIMENTS

Contract No. AT(11-1)-318

Issued May 6, 1957

Author- J. O. Brondel

ALCO PRODUCTS, INC.
P. O. Box 414
Schenectady, N. Y.

DISCLAIMER

DISCLAIMER

TABLE OF CONTENTS

I SUMMARY

On the basis of recent experience with APPR-1 fuel elements manufactured by ORNL, the following hot channel factors have been derived from the observed dimensional deviations:

Hot Channel Factor	Local	Average
Plate Spacing Deviation	1.0558	1.0230
Uranium Content Deviation	1.0100	1.0050
Meat Thickness Deviation	1.0650	1.0400
Clad Thickness Deviation	1.0051	1.0026

On the basis of recent zero power experiments at the Critical Facility and experience with the Flow Test Rig, both at Alco, the following Hot Channel Factors have been derived:

Water Gap Flux Peaking	1.25	1.00
Orifice Sizing	1.0332	1.0417

The combined Hot Channel Factor, a product of the individual factors, is therefore:

Combined Factor	1.4742	1.1167

II MAXIMUM DEVIATIONS CONSIDERED

Item	Nominal	Maximum Deviation
Internal Plate Spacing	.133"	-.003" average
		+.007" local
Uranium Content		+0.5% average
		+1.0% local
Meat Thickness	.020"	+.0008" average
		+.0013" local
Clad Thickness	.005"	+.0001" average
		-.0002" average
		±.0004" local
Relative Thermal Flux across Meat Portion of Fuel Plate	1.00	+25% average and local
Relative Channel Flow as Governed by Orifice Diameter	1.00	- 4% average and local

III DEFINITION OF AVERAGE AND LOCAL HOT CHANNEL FACTORS

In considering a particular dimensional or other type of deviation, the average hot channel factor is defined as the ratio of the highest possible coolant temperature rise to the corresponding nominal rise. As no axial heat flow in the fuel plates is considered, the average factor is influenced only by considerations affecting heat flow and weight flow rates.

The local hot channel factor is defined as the ratio of the highest possible local temperature gradient across the water film to the corresponding nominal gradient. The local factor is influenced only by considerations affecting heat flow rate and film coefficient.

It should be noted that while it is possible for the average channel factor to be simultaneously applicable along the entire length of the hottest channel, the local factor can only be applicable at a few, though entirely arbitrary, points along the path of flow.

The derivation of the following hot channel factors is based upon deviations of the stationary fuel elements only since factors derived for control rod elements are either equal or slightly smaller.

IV SYMBOLS AND NOMENCLATURE

A - Symbols

a - Max. negative average deviation of plate spacing, in.

A - Channel flow area, in.^{2}

b - Max. positive local deviation of plate spacing, in.

c - Specific heat of water, Btu/lb. - ^{o}F.

C - Flow coefficient

d - Nominal plate spacing, in.

D - Hydraulic diameter of channel, in.

f - Channel friction factor

F - Hot channel factor

h - Water film heat transfer coefficient, Btu/in.^{2} - ^{o}F. - sec.

k - Thermal conductivity, Btu - in./in.^{2} - ^{o}F. - sec.

L - Channel length, in.

p - Perimeter of channel, in.

Δ P - Pressure drop across channel, in. of water

q - Heat flow rate, Btu/in.^{2} - ^{o}F. - sec.

Q - Volumetric heat generation rate in meat, Btu/in.^{3} - sec.

t - Thickness of meat or clad in fuel element, in.

T - Temperature, ^{o}F.

TR - Bulk temperature rise of channel flow, ^{o}F.

μ - Water viscosity, lb./in. - sec.

V - Water velocity in channel, in./sec.

w - Width of channel, in.

W - Relative weight of uranium per unit volume of meat

x - Distance to point inside meat, measured from hot channel side perpendicular to plate, in.

ϕ - Thermal neutron flux

ρ - Water density, lb./in.^{3}

IV - SYMBOLS AND NOMENCLATURE (Cont'd)

B - Subscripts

a - Average conditions along length of channel

avg. - Average conditions across width of fuel plate meat

c - Clad

C - Contraction flow on entering channel

E - Expansion flow on leaving channel

f - Water film

h - Hot channel conditions

l - Local conditions along length of channel

m - Meat of fuel element

n - Nominal conditions

o - Outside channel (adjacent to hot channel)

p - Peak meat temperature

Δ T - Temperature gradient across water film

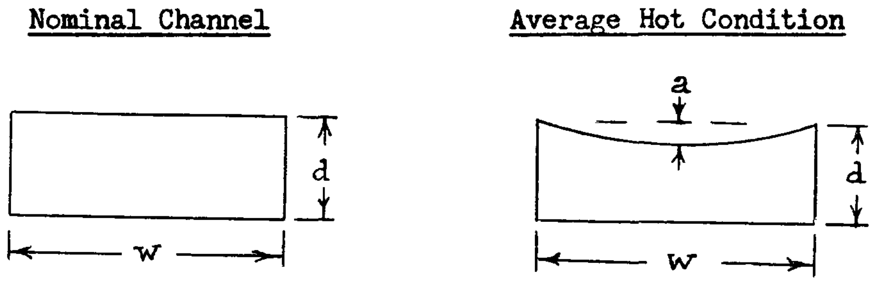
TR - Bulk temperature rise of channel flow

w - Water

V Plate Spacing Deviation Factors

A - Average Hot Channel Factor

A flow passage is restricted by plates bowing or sagging in the center. Deviations in the plate edge spacing are negligible.



The reduction in flow area is estimated at 2/3 of the center deviation times the channel width.

$$A_n = dw$$

$$A_{ha} = dw - \frac{2}{3}(a)\,w = (d - 2a/3)\,w$$

$$D_n = \frac{4\,A_n}{p_n} = \frac{4\,d\,w}{2\,(d + w)}$$

$$D_{ha} = \frac{4\,A_{ha}}{p_{ha}} = \frac{4\,(d - 2a/3)\,w}{2\,(d + w)}$$

As the pressure drop ΔP across each channel of the same element is essentially equal and as contraction and expansion coefficients for the channels are negligible, velocity variation is only a function of the average hydraulic diameter deviation.

$$\Delta P = \frac{V^2}{12\,(2g)}\left[C_C + 4\,f\,\frac{L}{D} + C_E\right]$$, where C_C and C_E are essentially zero.

$$\frac{V^2{}_n}{V^2{}_{ha}} = \frac{D_n}{D_{ha}} = \frac{d}{d - 2\,a/3}$$

$$F_{TR} = \frac{(TR)_h}{(TR)_n} = \frac{A_n}{A_{ha}} \; \frac{V_n}{V_{ha}} = \frac{d}{d-2a/3} \sqrt{\frac{d}{d-2a/3}} = \left[\frac{d}{d-2a/3}\right]^{3/2}$$

Substituting the numerical values applicable for the APPR-1:

d = .133 in.

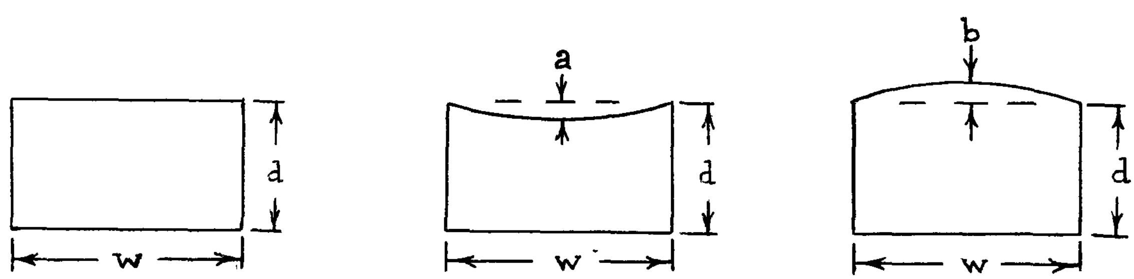
a = .003 in. (minimum average deviation)

F_{TR} = 1.0230

B - Local Hot Channel Factor

In defining the very worst possible channel thermally, the very conservative approach is made of considering that the average deviation could be the negative observed limit while a local deviation could simultaneously be at the positive observed limit in the same channel.

<u>Nominal Channel</u> <u>Average Hot Condition</u> <u>Local Hot Condition</u>



A_n = d w

A_{hl} = $d\,w + \frac{2}{3}\,(b)\,w = (d + 2\,b/3)\,w$

The local deviation is assumed to be sufficiently gradual to permit complete diffusion.

$$\frac{V_n}{V_{hl}} = \frac{V_n}{V_{ha}} \quad \frac{V_{ha}}{V_{hl}} = \sqrt{\frac{d}{d - 2a/3}} \cdot \frac{d + 2b/3}{d - 2a/3}$$

$$\frac{D_{hl}}{D_n} = \frac{A_{hl}}{A_n} = \frac{d + 2b/3}{d}$$

The film coefficient is reduced not only by the drop in coolant velocity but also by the increase in hydraulic diameter:

$$h = .023 \frac{K_w}{D} \left(\frac{\rho VD}{u}\right)^{0.8} \quad \left(\frac{c_w \; u}{K_w}\right)^{0.4}$$

$$F_{\Delta T} = \frac{T_h}{T_n} = \frac{h_n}{h_{hl}}$$

$$= \left(\frac{V_n}{V_{hl}}\right)^{0.8} \left(\frac{D_{hl}}{D_n}\right)^{0.2}$$

$$= \left(\sqrt{\frac{d}{d - 2a/3}} \cdot \frac{d + 2b/3}{d - 2a/3}\right)^{0.8} \left(\frac{d + 2b/3}{d}\right)^{0.2}$$

$$= \frac{d^{0.4} \; (d + 2b/3)}{d^{0.2} \; (d - 2a/3)^{1.2}} = \frac{d^{0.2} \; (d + 2b/3)}{(d - 2a/3)^{1.2}}$$

Substituting applicable numerical values for the APPR-1:

$b = .007$ in. (maximum local deviation)

$F_{\Delta T} = 1.0558$

VI Uranium Content Deviation

A - Average Hot Channel Factor

Both fuel elements forming the hot channel are assumed to contain the maximum uranium content per unit volume observed.

$$F_{TR} = \frac{TR_{ha}}{TR_n} = \frac{W_{ha}}{W_n}$$

The maximum positive fractional deviation of average uranium content is 0.5%.

$$F_{TR} = \frac{W_n \ (1 + .005)}{W_n} = 1.0050$$

B - Local Hot Channel Factor

$$F_{\Delta T} = \frac{\Delta T_h}{\Delta T_n} = \frac{W_{hl}}{W_n}$$

The maximum positive fractional deviation at local uranium content is 1.0%.

$$F_{\Delta T} = \frac{W_n \ (1 + .01)}{W_n} = 1.0100$$

VII Meat Thickness Deviation Factors

A - Average Hot Channel Factor

Both fuel elements forming the hot channel are assumed to contain the maximum meat thickness observed.

$$F_{TR} = \frac{TR_h}{TR_n} = \frac{t_{mha}}{t_{mn}}$$

The maximum positive dimensional deviation of average thickness is .0008 in.

$$F_{TR} = \frac{.020 + .0008}{.020} = 1.0400$$

B - Local Hot Channel Factor

$$F_{\Delta T} = \frac{\Delta T_h}{\Delta T_n} = \frac{t_{mhl}}{t_{mn}}$$

The maximum positive dimensional deviation of local meat thickness is .0013 in.

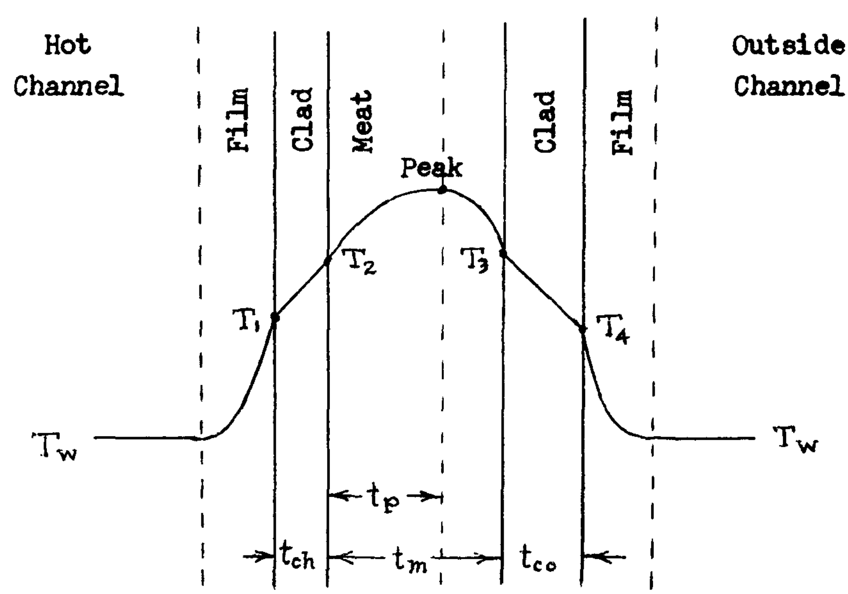
$$F_{\Delta T} = \frac{.020 + .0013}{.020} = 1.0650$$

VIII Clad Thickness Deviation Factors

A - Derivation of General Equations

If the two clad thicknesses of a fuel plate are unequal, a greater portion of the total heat generated in the meat will pass out through the thinner clad because of lower thermal resistance. A hot channel is, therefore, defined as being composed of two fuel plates whose inner and outer clad thicknesses are at the minimum and maximum observed values respectively. Both the average and local hot channel factors are then equal to the proportional increase in the heat transmitted through the inner sides.

A cross-section of the hot channel fuel plate is shown in the diagram below. The water temperatures are conservatively assumed equal.



The determination of hot channel factors is derived in the following steps.

a) Determine the general differential equation for the temperature distribution through the meat as a volume heat source.

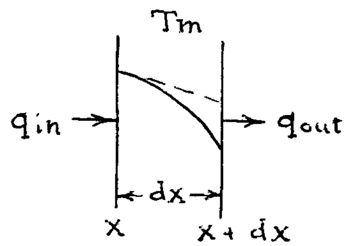
b) Determine the general solution for step (a). Substituting boundary conditions, solve for the general constants of integration.

c) Determine the location of the temperature peak by setting slope of temperature curve equal to zero. Portion the heat flow through two clads accordingly, in terms of meat boundary conditions.

d) Determine the temperature gradient through each clad and water film combination, using the results of step (c). The meat boundary conditions are then defined in terms of physical dimensions and properities.

e) Substitute the meat boundary conditions as determined by step (d) into the expressions for proportioning of heat flow as determined in step (c). The hot channel factor is then simply the ratio of the hot channel side flow to the average or nominal flow.

a) The general differential equation for temperature distribution through a volume heat source is determined as follows:



$$q_{in} = -k_m \left(\frac{dT_m}{dx}\right)_x$$

$$q_{out} = -k_m \left(\frac{dT_m}{dx}\right)_{x+dx}$$

$$q_{out} - q_{in} = Q\,dx \qquad \left(\frac{dT_m}{dx}\right)_{x+dx} = \left(\frac{dT_m}{dx}\right)_x + \left(\frac{d^2T_m}{dx^2}\right)dx$$

By proper substitution and simplification:

$$q_{out} - q_{in} = -k_m \left(\frac{dT_m}{dx}\right)_{x+dx} + k_m \left(\frac{dT_m}{dx}\right)_x$$

$$Q\,dx = -k_m \left[\left(\frac{dT_m}{dx}\right)_x + \left(\frac{d^2T_m}{dx^2}\right)dx\right] + k_m \left(\frac{dT_m}{dx}\right)_x$$

$$Q\,dx = -k_m \left(\frac{d^2T_m}{dx^2}\right)dx$$

$$\frac{d^2T_m}{dx^2} = -\frac{Q}{k_m}$$

b) The differential equation is solved by the reduction of order method:

$$\text{Let } \frac{dT_m}{dx} = y \; ; \quad \text{Then} \quad \frac{dy}{dx} = -\frac{Q}{k_m}$$

$$\int dy = -\frac{Q}{k_m}\int dx$$

$$\frac{dT_m}{dx} = -\frac{Q}{k_m}x + c_1$$

$$\int dT_m = -\frac{Q}{k_m}\int x\,dx + c_1\int dx$$

$$T_m = -\frac{Q}{2k_m}x^2 + c_1 x + c_2$$

Boundary conditions (See Fig. 1):

$$x = o \; ; \quad T_m = T_2$$

$$x = t_m \; ; \; T_m = T_3$$

By substitution of boundary conditions into the general solution:

$$T_2 = c_2$$

$$T_3 = - \frac{Q}{2K_m} t^2_m + c_1 t_m + T_2$$

$$c_1 = \frac{T_3 - T_2}{t_m} + \frac{Qt_m}{2K_m}$$

Substituting these results to obtain the specific solution:

$$T_m = - \frac{Q}{2K_m} x^2 + \left[\frac{T_3 - T_2}{t_m} + \frac{Qt_m}{2K_m}\right] x + T_2$$

c) To find the point of temperature peaking, and therefore the point of direction change in heat transfer, determine t_p where $\frac{dT_m}{dx} = o$:

$$\frac{dT_m}{dx} = - \frac{Q}{Km} t_p + \left[\frac{T_3 - T_2}{t_m} + \frac{Qt_m}{2K_m}\right] = 0$$

$$t_p = \frac{t_m}{2} + \frac{K_m}{Qt_m} \left(T_3 - T_2 \right)$$

It is seen from this expression for t_p that the peak temperature normally occurs in the center of the meat, but that a correction is necessary with unequal sink temperatures. This is, of course, as expected.

As heat generation per unit volume is uniform, the fraction of the total heat flow that is leaving the fuel plate on the hot channel side isequal to tp/tm. Therefore:

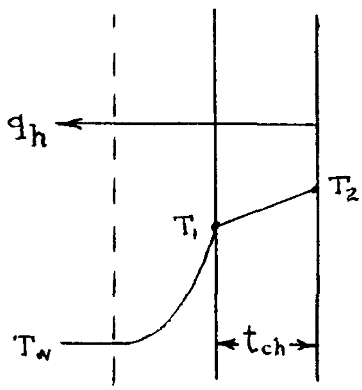
$$q_h = Q\,t_p$$

$$= \frac{Qt_m}{2} + \frac{K_m}{t_m}\left(T_3 - T_2 \right)$$

$$q_o = Q\,(t_m - t_p)$$

$$= \frac{Qt_m}{2} - \frac{K_m}{t_m}\left(T_3 - T_2 \right)$$

d) Since an expression is available for the heat flow rate through the clad and water film on either side, the corresponding temperature gradients can be determined:



$$T_2 - T_w = q_h\left(\frac{1}{h_{ch}} + \frac{1}{h_f}\right)$$

$$= q_h\left(\frac{t_{ch}}{K_c} + \frac{1}{h_f}\right)$$

Similarly for the other plate side:

$$T_3 - T_w = q_o\left(\frac{t_c}{K_c} + \frac{1}{h_f}\right)$$

By subtraction and substitution for q_o :

$$T_3 - T_2 = q_n\left[\frac{t_{co}}{k_c} + \frac{1}{h_f}\right] - q_h\left[\frac{t_{ch}}{k_c} + \frac{1}{h_f}\right]$$

$$T_3 - T_2 = \left[Qt_m - q_h\right]\left[\frac{t_{co}}{k_c} + \frac{1}{h_f}\right] - q_h\left[\frac{t_{ch}}{k_c} + \frac{1}{h_f}\right]$$

$$T_3 - T_2 = Qt_m\left[\frac{t_{co}}{k_c} + \frac{1}{h_f}\right] - q_h\left[\frac{t_{ch}}{k_c} + \frac{2}{h_f} + \frac{t_{co}}{k_c}\right]$$

For convenience in handling, this latter expression will be written as:

$$T_3 - T_2 = Qt_m \alpha - q_h \beta \; ; \quad \alpha = \frac{t_{co}}{K_c} + \frac{1}{h_f}$$

$$\beta = \frac{t_{co}}{K_c} + \frac{t_{ch}}{K_c} + \frac{2}{h_f}$$

e) The final expression for q_h can now be determined by substituting this final relationship in paragraph (d) for $T_3 - T_2$ into the equation for q_h in paragraph (c):

$$q_h = \frac{Qt_m}{2} + \frac{K_m}{t_m} (T_3 - T_2)$$

$$= \frac{Qt_m}{2} + \frac{K_m}{t_m} (Qt_m \alpha - q_h \beta)$$

$$= \frac{\frac{Qt_m}{2} + QK_m \alpha}{\frac{K_m}{t_m} \beta + 1}$$

The hot channel factor is now determined by dividing the maximum by the nominal heat flux

$$F = \frac{q_h}{q_n} = \frac{\dfrac{\frac{Qt_m}{2} + QK_m \alpha}{\frac{K_m}{t_m} \beta + 1}}{\dfrac{Qt_m}{2}}$$

$$F = \frac{\cancel{\phi} t_m + 2\cancel{\phi} k_m \alpha}{\cancel{2}} \cdot \frac{t_m}{K_m \beta + t_m} \cdot \frac{\cancel{2}}{\cancel{\phi} t_m}$$

$$F = \frac{t_m + 2K_m \alpha}{t_m + K_m \beta} \; ; \quad \alpha = \frac{t_{co}}{K_c} + \frac{1}{h_f}$$

$$\beta = \frac{t_{co}}{K_c} + \frac{t_{ch}}{K_c} + \frac{2}{h_f}$$

It is seen that when the two clad thicknesses t_{co} and t_{ch} are equal, then $\beta = 2\alpha$, and $F = 1$ as expected.

B - Average Hot Channel Factor

The following numerical values are applicable for the APPR-1 core:

t_m	=	.0200	in. (nominal)
t_{ch}	=	.0048	in. (minimum average)
t_{co}	=	.0051	in. (maximum average)
k_m	=	.000205	BTU/in. - ^oF - sec. (at 600 ^oF)
k_c	=	.000274	BTU/in. - ^oF -sec. (at 600 ^oF)
h_f	=	.00459	BTU/in.^{2} - ^oF - sec. (at rated flow velocity)

By computation:

α	=	236.0	in.^{2} - ^oF - sec./BTU
β	=	470.9	in.^{2} - ^oF - sec./BTU

Substituting into the expression for hot channel factors developed in section (a):

$$F_{TR} = \frac{TR_h}{TR_n} = \frac{q_h}{q_n} = \frac{t_m + 2K_m \alpha}{t_m + K_m \beta}$$

$F_{TR} = 1.0026$

c. Local Hot Channel Factor

The following numerical values are applicable for the local factor:

$t_{ch} = .0046$ in. (minimum local)

$t_{co} = .0054$ in. (maximum local)

By computation:

$\alpha = 237.1$ in.^{2} - ^oF - sec./BTU

$\beta = 471.3$ in.^{2} - ^oF - sec./BTU

Substituting into the expression for hot channel factors

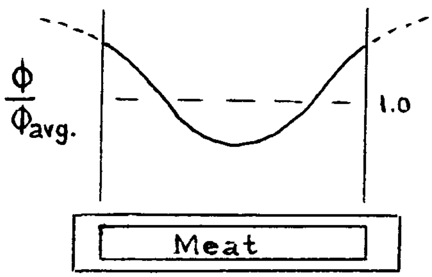
$$F_{\Delta T} = \frac{\Delta T_h}{\Delta T_n} = \frac{q_h}{q_n} = \frac{t_m + 2K_m \alpha}{t_m + K_m \beta}$$

$F_{\Delta T} = 1.0051$

IX WATER GAP FLUX PEAKING FACTORS

A. Average Hot Channel Factor

It has been determined from zero power experiments at the Alco Critical Facility that the thermal flux distribution across the meat width of a fuel plate surface is parabolic in nature.



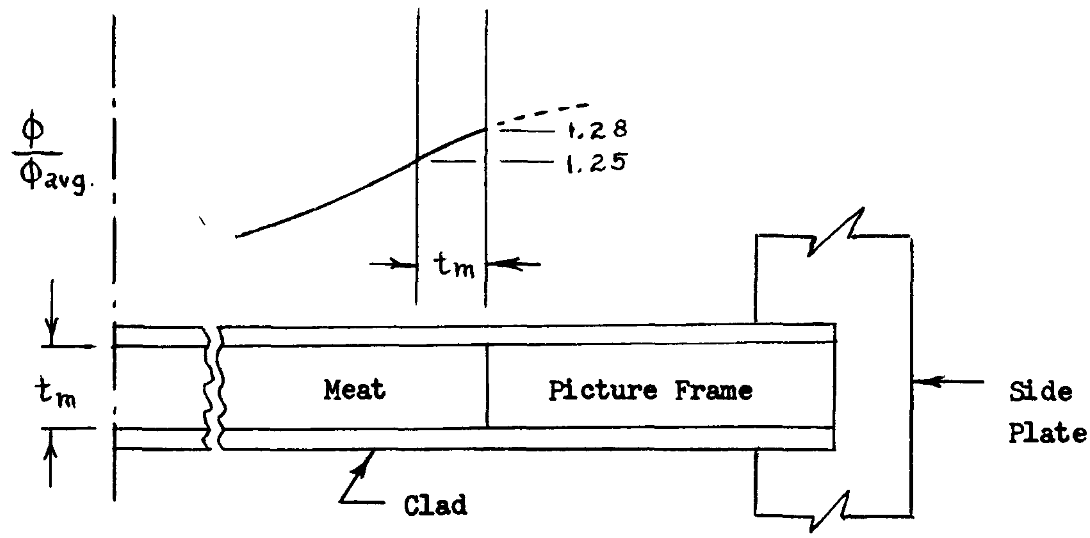
The water temperature rise up to any length along the channel is based upon progressive values of ϕ avg., or the average flux across the meat width. As the channel flow is well into the turbulent or high Reynolds number range, there are no local higher temperature rise regions in the main stream because of the continuous effective mixing process. The average hot channel factor is therefore unity.

$$F_{TR} = \frac{TR_h}{TR_n} = 1.00$$

B. Local Hot Channel Factor

Boundary layer or water film flow is essentially laminar and subject to varying local temperature gradients. The nominal gradient is based upon the average flux ϕ avg. across the width of the meat. As the film conductance is constant, the gradient is directly proportional to the local heat flow rate.

If heat flow is assumed to be always normal to a fuel plate surface, then the local hot channel factor is 1.28, as seen by the diagram below showing zero power experiment data.



However, such an assumption is unnecessarily conservative. As seen from the diagram, approximately to scale, a significant portion of the heat generated at the edges of the meat will flow outward through the picture frame. Previous temperature distributional studies for determining thermal stress indicate that the temperature along the clad surface will not rise above its value at a distance in from the meat edge equal to the meat thickness. Assuming normal-to-surface heat flow at this latter location, the highest local hot channel factor is then 1.25.

$$F_{\Delta T} = \frac{\Delta T_h}{\Delta T_n} = 1.25$$

X ORIFICE SIZING FACTORS

A. Average Hot Channel Factor

The orifice diameter required to yield a given flow rate for a particular fuel element was determined experimentally using the Alco Flow Test Rig. Some flow deviation allowance must be made for instrument error and experimental accuracy. As the calculated value of the proper orifice diameter is expressed to the nearest 1/64" for machining purposes, an additional deviation allowance must be introduced. Experience with the APPR-1 demonstrates that a combined maximum deviation of ±4% is sufficient for both stationary element and control rod flow.

The hot channel factors associated with a 4% reduction in flow from an undersized orifice are determined in a manner similar to that used for the factors associated with plate spacing deviation with the exception that no change in the channel flow area is considered.

$$F_{TR} = \frac{TR_h}{TR_n} = \frac{V_n}{V_h}$$

$$F_{TR} = \frac{V_n}{V_n(1 - .04)} = 1.0417$$

B. Local Hot Channel Factor

$$F_{\Delta T} = \frac{\Delta T_h}{\Delta T_n} = \frac{h_n}{h_h} = \left(\frac{V_n}{V_h}\right)^{0.8}$$

$$F_{\Delta T} = \left[\frac{V_n}{V_n(1 - .04)}\right]^{0.8} = 1.0332$$